The Capital Asset Pricing Model (CAPM)

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- We have so far studied the relevant portfolio opportunity set (mean-variance efficient portfolios)
- We now study more specifically portfolio demand, for a given supply of stocks
- Key question: *If everyone holds efficient portfolios, what must stock prices be to clear the market (i.e., results in 100% of the supply of shares being held by investors)*

Assumption 1: Perfect markets

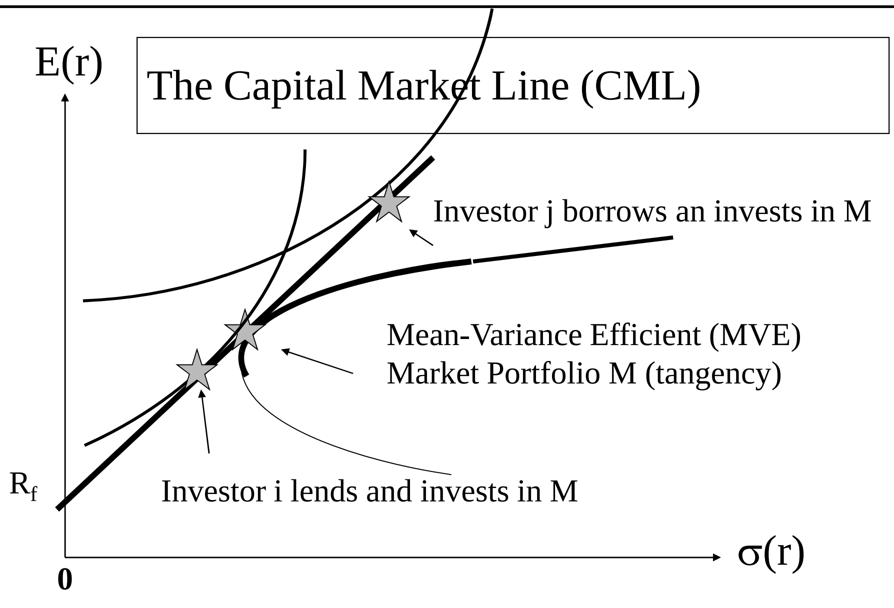
- Zero transactions costs
- All assets (stocks, bonds, human capital, real estate, etc.) are tradable in perfectly divisible amounts
- Zero taxes
- Competition prevents any individual from affecting security prices
- Unlimited short sales and borrowing and lending at the risk-free rate r_f

Assumption 2: Rational investors

- Investors have a *one-period horizon* and preferences over the mean (E) and return variance (σ^2) only.
  - I.e., they have quadratic utility or, alternatively, *returns are jointly normally distributed*
- Investors have *homogenous* and *rational expectations*.
  - *Thus, the location of the MVE frontier is the same for all investors*
- Investors may have different risk tolerances.
  - Thus, they may hold different combinations of the risk-free asset and the risky portfolio

Implications

- Every investor solves the passive portfolio problem studied earlier, and thus holds a combination of the risk-free asset and the tangency portfolio on the MVE frontier
- Since everyone draws the same CAL (homogenous expectations), *everyone also holds the same tangency portfolio*
- *Since all securities must be held by someone in equilibrium this tangency portfolio must be the "market" portfolio M of all assets*



- M is a portfolio of <u>all</u> risky securities held in proportion to their market value:

  x_{iM}= (i's value)/(total value of all securities)
- The expression for the CML: Form a portfolio of M and the risk-free asset F:

  $E_p = x_F r_F + (1-x_F)E_M$

  $\sigma_p = (1-x_F)\,\sigma_M$

  $\rightarrow 1-x_F = \sigma_p/\sigma_M$

  $\rightarrow \boxed{E(r_p) = r_F + [(E(r_M) - r_F)/\sigma_M]\sigma_p}$ (CML)

- The CML is the locus of all MV efficient portfolios.
  - Thus, this equation prices all <u>efficient</u> portfolios only
  - Since individual assets are MV inefficient, the CML does <u>not</u> provide a pricing equation for individual assets
- Recall that for MV efficient portfolios, the portfolio's variance, σ^2_p , consists of systematic (nondiversifiable) risk only
  - This is not true for individual securities and inefficient portfolios

- Recall from earlier that security i's marginal contribution to the risk of portfolio p is given by the covariance σ_{ip} (multiplied by the weight x_i)
- Let's standardize this covariance by the total variance of the portfolio, and let's focus on portfolio M:

$$\rightarrow \beta_{iM} \equiv \sigma_{iM}/\sigma^2_M$$

- Intuitively, this "beta" risk must what the market rewards investors for carrying
  - You can costlessly get rid of the remaining unsystematic risk of the security by placing it in a large (efficient) portfolio like M

- We want a pricing model that relates a security's beta risk to the market price per unit of beta risk
  - This is precisely what the CAPM does
- <u>Note that the market price per unit of beta-risk must be the same for all securities</u>
  - Also referred to as the "law of one price", or a "no-arbitrage" condition
  - In a CAPM world, it implies that two stocks with the same amount of beta-risk must have the same expected returns
- Mathematically, you derive the CAPM pricing equation by equating the slope of the CML and the MV frontier at M

$$E_i = r_F + [(E_M - r_F)/\sigma^2_M]\sigma_{iM}$$

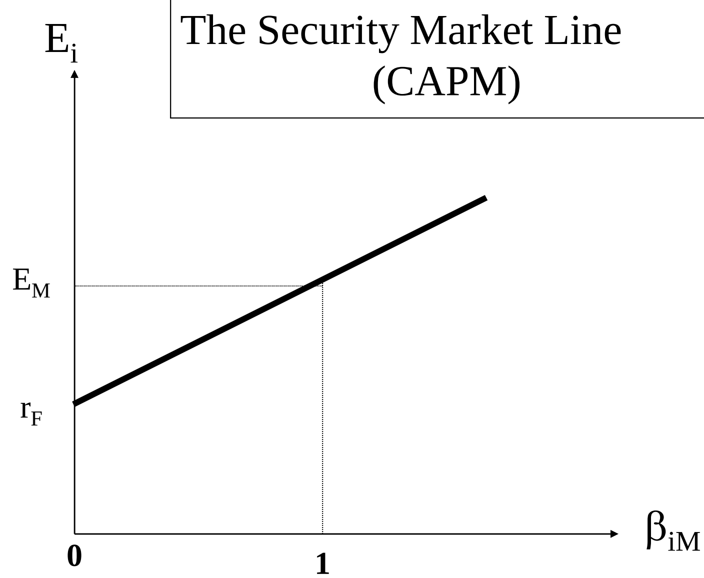
or

$$\boxed{E_i = r_F + \beta_{iM}(E_M - r_F)}$$

Security Market Line or CAPM

where, as before, $\beta_{iM} = \sigma_{iM}/\sigma^2_M$

The Security Market Line (CAPM)



- All securities lie on the Security Market Line. It is thus a pricing equation for any individual security

- Rewrite the *CAPM*:

$$E_i = (1-\beta_{iM})r_F + \beta_{iM}E_M$$

  - Since $(1-\beta_{iM}) + \beta_{iM} = 1$, the right-hand side is a portfolio where you invest the proportion $(1-\beta_{iM})$ in the risk-free asset F and β_{iM} in the market M

- Thus, in the CAPM, security i's expected return simply equals the expected return on an efficient benchmark portfolio with the same systematic risk β_{iM}

- Note:
  - You can in principle use any efficient portfolio as a benchmark portfolio.
  - It works with M since the CAPM implies that M is MV efficient.
  - Recall that the model requires M to include all assets in the universe. Thus, it is fundamentally not a testable concept.
  - Empirical tests instead asks whether the CAPM "works well" and "better" than another model

Beta as a regression coefficient

- Consider the time-series regression model:

  $r_{it}-r_{Ft} = \alpha_i + \beta_i(r_{Mt}-r_{Ft}) + \varepsilon_{it}$

- The OLS regression coefficient is

  $\beta_i = \sigma_{iM}/\sigma^2_M$ (assumes $E_\varepsilon=0$, $cov(r_M,\varepsilon_i)=0$) which is identical to our earlier beta definition

- The variance of the regression equation:

  $\sigma^2_i = \beta^2_i\sigma^2_M + \sigma^2_\varepsilon$

Total risk=systematic risk+unsystematic risk

Relaxing the CAPM assumptions

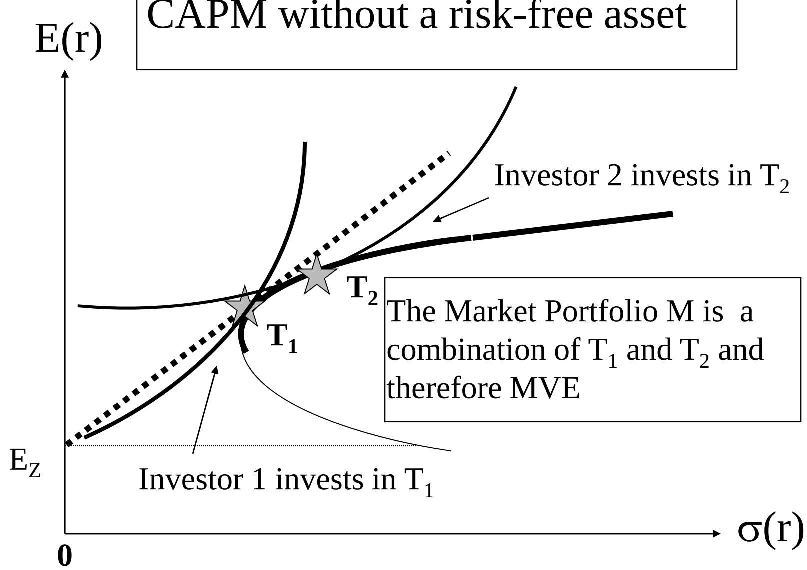
- "What if"
  - there are no risk-free assets, or
  - investors have mulitperiod investment horizons, or
  - investors have heterogeneous expectations?
- For each of these complications, check whether the market portfolio M still mean-variance efficient

What if no risk-free asset?

- Even a risk-free bond is "risk-free" only if you hold it to maturity
  - If the risk-free rate of return changes over time, selling the bond before maturity produces an uncertain value.
- So what if there is no risk-free bonds with the maturity that you want?
  - In the CAPM, any asset or portfolio is "risk-free" as long as it has a zero beta with M
  - Let Z be such a "zero-beta" portfolio

Eckbo (26) 17

CAPM without a risk-free asset



Eckbo (26) 18

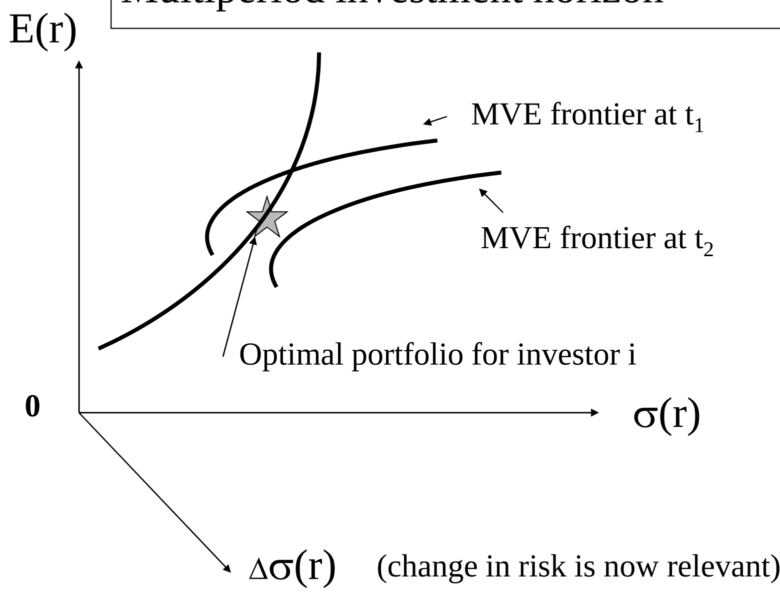
- Notice:
  - Each investor now invests in a MVE tangency portfolio T
  - Since a combination of MVE portfolios (with positive weights) is itself MVE, the market portfolio M is still MVE
  - Consequently, the CAPM holds, but with the portfolio Z (which is not MVE) acting as the risk-free asset:

$$E_i = E_Z + \beta_{iM} (E_M - E_Z)$$

What if multiperiod horizon?

- CAPM assumes investors have a one-period horizon: invest today and consume returns tomorrow (static model)
  - If investors instead form portfolios based on a multiperiod time horizon, they may worry about stochastic changes in the investment and consumption opportunity sets over time
  - Example: If you are a heavy consumer of corn, you may want to form a portfolio more heavily weighted towards corn-farm stocks in order to hedge your purchasing power for corn in the future

Multiperiod investment horizon

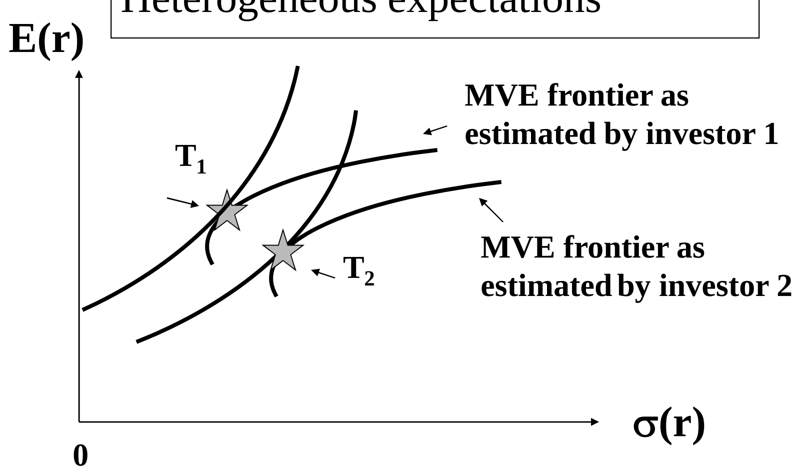


- Investor are still forming efficient portfolios
  - However, in Figure 4, their portfolios are efficient in the three-dimensional space (E, σ, Δσ). This efficient portfolio is no longer efficient in the (E, σ)-space alone.
  - As a result, the market portfolio M is also not MVE and the CAPM does not hold
  - The correct model is one where the new hedge factor (here Δσ) is added to the model with its own factor loading or "beta"

What if investors have heterogeneous expectations?

- Now each investor draws his or her own location for the MV efficient frontier
- Each investor i finds his or her optimal risky portfolio as the tangency portfolio T_i
- As always, the market portfolio M is the value-weighted portfolio of all the individual T_i's
- Since there is no "one" MVE frontier, M is no longer MVE, and the CAPM does not hold

Heterogeneous expectations



→ **The market portfolio M is not MVE**

Nontradable assets: What about human capital (HC)?

- In the CAPM, the market portfolio contains <u>all</u> assets – including your human capital
- But, in practice, HC is nontradable (slavery is forbidden)
- Also, for many, HC is the largest risky asset in their individual portfolio
- CAPM breaks down as the market portfolio is no longer efficient
  - Need to take into account the covariance between HC and the tradable assets
  - Creates a second pricing factor

Conclusion

- CAPM is a simple, static pricing model making strong assumptions
- It provides important insights about priced risk in equilibrium
  - However, empirical evidence suggests that the usual stock market proxy for M is not MVE
  - The true asset pricing model likely to have more than a single pricing factor
  - Will therefore turn to multifactor, APT models